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Explaining Credit Default Swap Spreads by Means of Realized Jumps and Volatilities in the Energy Market

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Abstract

This paper studies the relationship between credit default swap (CDS) spreads for the Energy sector and oil futures dynamics. Using data on light sweet crude oil futures from 2004 to 2013, which contains a crisis period, we examine the importance of volatility and jumps extracted from the futures in explaining CDS spread changes. The analysis is performed at an index level and by rating group; as well as for the pre-crisis, crisis and post-crisis periods. Our findings are consistent with Merton’s theoretical framework. At an index level, futures jumps are important when explaining CDS spread changes, with negative jumps having higher impact during the crisis. The continuous volatility part is significant and positive, indicating that futures volatility conveys relevant information for the CDS market. As for the analysis per rating group, negative jumps have an increasing importance as the credit rating deteriorates and during the crisis period, while the results for positive jumps and futures volatility are mixed. Overall, the relation between the CDS market and the futures market is stronger during volatile periods and strengthened after the Global Financial Crisis.

JEL Classification: G12, G13, C14

Keywords: Oil futures, CDS spread, realized jumps, realized volatility

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1 Introduction

The seminal work of Merton (1974) investigates the intrinsic relationship between credit risk, equity volatility and equity returns. It underpins the negative correlation between stock movements and credit risk, as well as the positive correlation between stock volatility and credit risk. Following this work, many empirical studies have analyzed the interactions between these three quantities. Most of them focus on finding the determinants of credit risk using several financial variables, including stock volatility and stock returns. These variables are confirmed to be important, which is consistent with Merton’s intuition; see among others Collin-Dufresne et al. (2001) and Campbell and Taksler (2003). Credit risk was initially measured by bond yield spread while equity volatility was obtained by using mean squared log-returns. The evolution of financial markets led to a reassessment of this intrinsic relationship between credit risk, equity volatility and equity returns. In particular, the rise of the credit default swap (CDS) market has provided an alternative way of quantifying credit risk whereas options provide a more forward looking point of view of equity volatility; see Benkert (2004) and Ericsson et al. (2009).

During the past decade equity volatility has become an asset class by itself, with the VIX being the most prominent example. Nowadays, volatility derivatives, including simple futures contracts as well as more sophisticated option contracts written on volatility indices, are actively traded. In the credit market, the credit default swap (CDS) serves as the underlying asset for options, which include collateralized debt obligations (CDOs) and other complex products (first-to-default, N-to-default and alike). The close relationship between these markets was strikingly illustrated during the Global Financial Crisis (GFC). It also led to many research works on contagion and market linkages.

The increasing complexity of financial derivatives and their interconnection urges the need for a consistent modeling framework for credit risk, stock return risk and stock volatility risk. So far, only a few research papers have addressed this important task. These include Carr and Wu (2007) who propose a model for joint dynamics of the stock with stochastic volatility and credit default intensity\(^1\) that allows for a consistent pricing of stock options and CDS. The authors

\(^1\)In the first order approximation CDS spread can be approximated by credit default intensity.
estimate the model by calibrating simultaneously time series of options and credit default swap spreads. They obtain rather puzzling results suggesting that default intensity is negative, and that there is a negative correlation between stock volatility and default intensity. Both results are inconsistent with Merton’s framework. We note that the authors link the default intensity and the stock dynamics through the stock jump intensity (i.e. the stock jump arrival rate affects the default intensity) but the stock jump itself has no impact on the CDS market. A slightly different model was later proposed in Carr and Wu (2010) but it also leads to puzzling conclusions. Indeed, the authors find a zero correlation between default intensity and stock volatility, and a closer look at their results leads to an even more problematic conclusion as they find that stock/volatility dynamics are completely unrelated to the CDS intensity. Thus, a complete contradiction with Merton’s results.

These results are also problematic from a practical point of view as equity derivatives and more precisely put options are often used as a credit derivatives. Along that line, let us mention the works of Cao et al. (2011) and Carr and Wu (2011) among many others. The lack of a consistent pricing framework for both credit and equity risks therefore results in failure to manage the financial risk appropriately; and taking into account the size of these markets, it raises the question of the overall derivative market stability.

In light of these results it appears to us that the definition of a consistent model for the stock market, the volatility market and CDS market has not yet been established, and the results obtained so far might be either due to a model misspecification and/or numerical/computational difficulties arising when implementing the models. In fact, both aforementioned papers (Carr and Wu (2007, 2010)) involve challenging numerical algorithms that can jeopardize the estimation procedure. Regarding model specification problems, both papers establish the link between the stock-volatility market and CDS market through the relationship between the stock jump intensity (i.e. stock jump arrival rate) and the default intensity, rather than linking the stock jump size to the default intensity. Thus, maybe linkages between these two markets should be performed through jump sizes rather than jump intensity or jump times. Financial asset jump activity has always been considered as an important component of asset dynamics; see Merton (1976) or Bates (2000). Availability of high frequency data along with econometric works (see
e.g. Aït-Sahalia (2002, 2004), Barndorff-Nielsen and Shephard (2004, 2006)) have enabled further insight into investigation of the price components. The results of Carr and Wu (2007, 2010) could possibly be explained by an underestimation of the role of jumps in market linkages as none of these models exhibit this property.

The existing literature provides a convenient framework to analyze the joint dynamics of stock returns, stock volatility and credit risk along with the role of stock jump activity. Indeed, Zhang et al. (2009) examine to what extent equity jumps and volatility are able to explain CDS spreads. Performing a panel analysis on a sample ranging from January 2001 to December 2003 for 307 U.S. firms, the authors find that equity volatility given by the bipower variation is an important determinant of CDS spread changes. Beyond stock returns, stock volatility and stock jump activity, they also consider macro-variables and accounting variables. Surprisingly, these authors find that equity jumps, either positive or negative, as well as jump intensity and jump volatility, are not significant when explaining CDS spread changes. Although they conclude that jumps are irrelevant the considered dataset, their framework, restricted to CDS spread, stock volatility and stock jumps, can be used to evaluate how market linkages depend on jump activity.

Our work contributes to the literature by introducing the first comprehensive analysis of the relationship between CDS spread for the Energy sector and oil futures jump and volatility activities. As opposed to previous studies performed for equity markets, our sample is large, ranging from January 2004 to December 2013, and contains both low and high volatility periods allowing to determine how market conditions affect this relationship. We perform the analysis for the CDS spread at an index level and per rating group, thus assessing the impact of creditworthiness on this relationship. We also split the sample into pre-crisis, crisis and post-crisis periods. Our results are consistent with Merton’s theoretical framework. At an index level, we find that futures jumps are an important ingredient when explaining CDS spread changes, with negative

\[^2\text{In the following we will be also referring to Tauchen and Zhou (2011) and Wright and Zhou (2009) who investigate the explanatory power of jumps, either for bond yield spreads or bond excess returns, but do not consider the CDS market. Notice also the works on WTI crude oil futures contracts by Sévi (2014, 2015), with the former underlining the role of jumps to forecast volatility while the latter focuses on convenience yield. The work of Naifar (2012) considers the relationship between CDS spreads and jumps but within a very different mathematical framework, namely, using copulas.}\]
jump components having higher impact during the crisis period. Furthermore, we find that the continuous volatility part is significant and positive indicating that futures volatility conveys relevant information for the CDS market. As for the results per rating group, we find that negative jumps have an increasing importance as the credit quality deteriorates, as well as during the crisis period. We observe mixed results for the significance of positive jumps and futures volatility when looking across different rating categories and various sample periods. Overall, the relation between the CDS market and the futures market appears to be stronger during volatile market conditions and strengthens after the GFC.

The paper is organized as follows. We present the key ingredients for the jump detection framework in Section 2. A description of the empirical data used in our analysis is provided in Section 3. Regression tests and analysis are performed in Section 4. Section 5 provides consistent model perspectives and Section 6 concludes the paper.

2 Model Specification

Let \( s_t = \ln(S_t) \) be the log-asset price whose dynamics evolve under the influence of a jump-diffusion process

\[
\text{d}s_t = \mu_t\text{d}t + \sigma_t\text{d}W_t + J_t\text{d}q_t, \tag{1}
\]

where \( \mu_t \) and \( \sigma_t \) are the instantaneous drift and diffusion terms of the return process, respectively; \( J_t \) is the log jump size with mean \( \mu_J \) and standard deviation \( \sigma_J \), \( W_t \) is a standard Brownian motion and \( dq_t \) is a Poisson process with intensity \( \lambda_J \). Time is measured in daily units and we define the intraday returns as

\[
r_{t,i} = s_{t,i}\Delta - s_{t,(i-1)}\Delta, \tag{2}
\]

where \( r_{t,i} \) refers to the \( i^{th} \) within-day return on day \( t \), with \( \Delta \) being the sampling frequency within each day such that \( m = 1/\Delta \) observations occur every day and as \( \Delta \to 0 \) we have that \( m \to \infty \).

As mentioned above, jump diffusion models have a long history in finance. More specific to the commodity market, recent works underpin the importance of jumps (see Larsson and Nossman (2011), Chevallier and Ielpo (2012) and Brooks and Prokopczuk (2013)); find that intensities for commodity price jumps are time varying (refer to Diewald et al. (2015)); and underline that jumps have important consequences in risk management (Chen et al. (2013), when considering jumps as a modeling strategy for extreme events).
Barndorff-Nielsen and Shephard (2004) propose two measures for quadratic variation process namely, the realized variance ($RV$) and the realized bipower variation ($BV$) that converge uniformly as $\Delta \to 0$ to different quantities of the jump diffusion process such as

$$RV_t = \sum_{i=1}^{m} r_{t,i}^2 \to \int_{t-1}^{t} \sigma_s^2 ds + \int_{t-1}^{t} J_s^2 dq_s, \quad (3)$$

$$BV_t = \frac{\pi}{2} m \sum_{i=2}^{m} |r_{t,i}| |r_{t,i-1}| \to \int_{t-1}^{t} \sigma_s^2 ds. \quad (4)$$

As it is evident from Eq.(3) and Eq.(4), the difference between the realized variance and the realized bipower variation is zero when there is no jump and strictly positive when there is a jump. For detecting jumps, we adopt the ratio test, proposed in Huang and Tauchen (2005) and Andersen et al. (2007), where the test statistic

$$RJ_t \equiv \frac{RV_t - BV_t}{RV_t} \quad (5)$$

is an indicator for the contribution of jumps to the total within-day variance of the process. This test statistic converges in distribution to a standard normal distribution when using an appropriate scaling

$$ZJ_t = \frac{RJ_t}{\sqrt{\left\{ \left( \frac{\pi}{2} \right)^2 + \pi - 5 \right\} \Delta \max \left( 1, \frac{TP_t}{BV_t^2} \right)}} \to N(0,1). \quad (6)$$

In Eq.(6) $TP_t$ is the tripower quarticity that is robust to jumps; it is defined in Barndorff-Nielsen and Shephard (2004) as

$$TP_t = m \mu_{4/3}^{-3} m \sum_{i=3}^{m} |r_{t,i-2}|^{4/3} |r_{t,i-1}|^{4/3} |r_{t,i}|^{4/3} \to \int_{t-1}^{t} \sigma_s^4 ds, \quad (7)$$

where

$$\mu_k = 2^{k/2} \frac{\Gamma \left( (k + 1)/2 \right)}{\Gamma \left( 1/2 \right)}, \quad k > 0.$$ 

Assuming that there is at most one jump per day and that jump size dominates the return when a jump occurs (Andersen et al. (2007)), daily realized jump sizes can be obtained as

$$\hat{J}_t = \text{sign}(r_t) \times \sqrt{(RV_t - BV_t)} \times I_{(ZJ_t \geq \Phi_\alpha^{-1})}, \quad (8)$$
where \( \Phi(\cdot) \) is the cumulative standard normal distribution function with \( \alpha \) being the level of significance and \( I_{(Z_J \geq \Phi^{-1}_\alpha)} \) is an indicator function which takes the value of one if there is a jump on a given day, and zero otherwise.\(^4\)

Once the realized jumps have been established, we can compute the jump mean \( \hat{\mu}_J \), the variance \( \hat{\sigma}_J \) and intensity \( \hat{\lambda}_J \) as follows

\[
\hat{\mu}_J = \text{Mean of } J_t, \tag{9}
\]
\[
\hat{\sigma}_J = \text{Standard deviation of } J_t, \tag{10}
\]
\[
\hat{\lambda}_J = \frac{\text{Number of jump days}}{\text{Number of trading days}}, \tag{11}
\]

It has been shown in Tauchen and Zhou (2011) that such an approach for estimation of realized jump parameters is robust with respect to drift and diffusion function specifications. It makes easy to specify the jump arrival rate, avoiding elaborate estimation methods, and yields reliable results under various settings, for instance, when the sample size is either finite, increasing or shrinking. The methodology developed in Tauchen and Zhou (2011) serves as framework for Wright and Zhou (2009) and Zhang et al. (2009).

3 Data Description

In this study we consider both the CDS market and the commodity market during the period from January 2004 to December 2013. As the financial markets went through very different behaviors during this time, we found it instructive to split the selected period into three sub-samples: the first runs from January 2004 to December 2007 and will be qualified as the pre-crisis period; the second spreads from January 2008 to end of 2009, covering the GFC, and is named the crisis period\(^5\); the third and last sub-sample goes from January 2010 to December 2013 and

\(^4\)There exist different methodologies for jump detection, and Dumitru and Urga (2012) discuss implications of the chosen methodology on the output (jumps). The proposed methodology that detects jumps by comparing \( RV_t \) with \( BV_t \) leads to a more conservative size of the test, and is regarded as a good choice by Dumitru and Urga (2012) if the sample period is characterized by high volatility. Since we consider the time period that contains the GFC, we resort to this alternative.

\(^5\)It is usually agreed that the start of the GFC took place at the end of July 2007 but for the Energy CDS market the surge of CDS spread occurred slightly later. Selecting December 2009 for the “end” of the GFC might be very surprising at first sight. This choice is mainly motivated by two factors. First, around that date the CDS
is referred to as the post-crisis period. For each market we provide details that include the data source, data processing and descriptive statistics. We first focus on the energy market and then discuss the credit default swap market.

3.1 Futures Market

For the energy market we consider the front running light sweet crude oil futures (i.e., the futures with the shortest maturity) quoted on New York Mercantile Exchange (NYMEX) since it is one of the most traded futures in the energy sector. Because of its liquidity and importance it has been used in many previous empirical studies (see Askara and Krichene (2008), Souček (2013) and Chevallier and Sévi (2012) among others). High frequency data from January 2004 to December 2013 is obtained from SIRCA\textsuperscript{6}. We restrict the computations to 5-minute interval quotes from 9:30 am to 3:30 pm as it is well known that this sampling frequency avoids microstructure noise effects that can cause biases in the estimation of the realized volatility. We compute daily realized volatility ($RV_t$) and realized bipower variation ($BV_t$) using Eq.(3) and (4), respectively. From these two quantities we extract daily jumps using Eq.(8) and split the resulting time series into positive and negative parts that will be denoted as $J_t^+$ and $J_t^-$, respectively. Following the literature, see Zhang et al. (2009) and Tauchen and Zhou (2011) among others, we will average the data but in contrast to the literature we will do so only fortnightly, from Thursday to Wednesday (i.e., Wednesday two weeks later) and keep only these two-week spaced averaged values (further details will be provided later). For simplicity, we use the same notation for $RV_t$, $BV_t$, $J_t^+$ and $J_t^-$ for these fortnightly observations. Thus, our sample ranges from 21 January 2004\textsuperscript{7} to 18 December 2013, with fortnightly spaced observations. Figure 1 shows daily annualized realized volatility time series for the futures with the resulting $BV_t$ and $J_t$ series presented in Figure 2 and 3, respectively. From Figure 3 we note that both negative and positive jumps are prevalent throughout the entire period under consideration.

\[ \text{[ Insert Figure 1 here]} \]

\textsuperscript{6}http://www.sirca.org.au/

\textsuperscript{7}Averaging from Thursday, 8 January to Wednesday, 21 January.
We also compute the mean jump intensities for both the positive and negative jumps and denote these quantities as $\lambda^+$ and $\lambda^-$, respectively. We report in Table 1 the descriptive statistics for the differences in bipower variation $\Delta BV_t$, the jumps $J^+_t$ and $J^-_t$, as they will be used in the regression analysis performed in the next section, as well as the level of $BV_t$ as it allows for more intuitive interpretation.

Over the entire sample the mean value for the realized bipower variation ($BV_t$) corresponds to $3.6 \times 10^{-4}$ and when computed over the three different sub-samples, pre-crisis, crisis and post-crisis periods, it gives $3.22 \times 10^{-4}$, $7.86 \times 10^{-4}$ and $1.89 \times 10^{-4}$, respectively. The discrepancies between these values and the peak reached during the crisis period illustrate the impact of the GFC that started on the futures market through a substantial increase of futures volatility. The GFC impact is also well pronounced when looking at the standard deviation for the $BV_t$, which is higher during the crisis period compared to either pre- or post-crisis. Note also that the post-crisis value is smaller than the pre-crisis value, which is related to the influence of jumps as discussed below. Regarding the bipower variation change, given by $\Delta BV_t$, it is large and positive during the crisis period, implying an increase in the continuous volatility component during that critical period, while for the two other sub-samples the mean values are negative. Regarding the standard deviations, whether we consider the level $BV_t$ or the change $\Delta BV_t$, the values corresponding to the crisis period are substantially larger by a multiple factor of approximately 3 for the changes and 5 for the level. The discrepancies observed for the statistics computed over the different sub-samples justify the partitioning of the sample.

As for the jumps, positive and negative parts have approximately the same magnitude (means are close in absolute values), as well as similar standard deviations, when computed over the entire sample period. For the three sub-samples, positive and negative jumps display large variability. During the crisis period the mean values (in absolute value terms) and standard deviations are at least twice as high as those obtained for the pre-crisis period; and are also
significantly larger compared to the post-crisis period. Furthermore, post-crisis mean values are larger than their pre-crisis counterparts, which is well pronounced for negative jumps (with an increase of 63%), while for positive jumps, an increase of 13% is observed. This fact should be put in perspective with the remark made for $BV_t$, the continuous component of the realized volatility. These findings suggest that the importance of jumps has increased after the crisis period. Lastly, the mean jump intensities, which represent the average number of jumps per unit of time, characterize the jump activity and complete the statistics already presented. For the negative jumps the highest intensity is achieved during the post-crisis period, a result surprising at first as we would expect that the largest value should be observed during the crisis period. However, let us consider this value in perspective with other properties of negative jumps. Compared to the post-crisis period, negative jumps occurring during the crisis period are slightly less frequent (22% against 28% per year) but are 1.65 times larger in magnitude and also exhibit a greater variability (the standard deviation is at least twice as high as that during the post-crisis period). In contrast, post-crisis negative jumps are more frequent but of a smaller size. As for the pre-crisis period, the intensity and the jump size of negative jumps are 50% and 38% smaller compared to those for the post-crisis period, respectively. For the standard deviations, the results are as expected; they are at least twice as large during the crisis period compared to the pre- or post-crisis periods. If we restrict our consideration to the first two moments and the jump intensity, positive jumps exhibit similar qualitative statistical properties to those of negative jumps as all the remarks made for the latter apply to the positive jumps as well, but appear to be less pronounced.

3.2 CDS Market

A CDS is a credit derivative contract between two counterparties that essentially provides insurance against the default of an underlying. In a CDS, the protection buyer makes periodic payments to the protection seller until the occurrence of a credit event or the maturity date of the contract, whichever comes first. The premium paid by the buyer is quoted as an annualized spread, measured in basis points (bps), and referred to as the CDS spread. If a credit event (default) occurs on the underlying financial instrument (a bond from a specified set of bonds, all of them having some precise characteristics), the buyer is compensated for the loss incurred.

\(^8\)In our case these values are the mean number of jumps per year.
as a result of the credit event, receiving the difference between the par-value of the bond and its market value after default.

Our dataset uses CDSs on corporate bonds collected on a daily basis from Markit. We restrict our analysis to the 5-year maturity, which is considered to be the most liquid, from January 2004 to December 2013. We take non-sovereign entities from the Energy sector (previously named Oil & Gas)\(^9\). The CDSs are written on senior unsecured debt (RED tier code: SNRFOR) and denominated in USD. We average the individual CDS spreads using an equal weighting scheme to produce a 5-year CDS index value for this sector (the average number of entities is 44.8). In order to be consistent with the volatility data, the CDS time series are sampled fortnightly with the first and last observations corresponding to 21 January 2004 and 18 December 2013, respectively\(^{10}\).

Table 1 reports means and standard deviations for the CDS level and change for the entire sample as well as for the three sub-samples. For the entire sample, the mean CDS level is 139.73 and the corresponding standard deviation is 61.97. The mean CDS values for the pre-crisis, crisis and post-crisis periods are 105.00, 221.38 and 133.76, respectively. The historical high CDS spread levels observed during the GFC is consistent with the tremendous uncertainty taking place in the financial markets at that time. Furthermore, the standard deviation during that period is 79.82, which is four times higher than the standard deviation during the pre-crisis period and twice as high as that for the post-crisis period. Note also that the CDS market shows a greater level of uncertainty for the CDS spreads during the post-crisis period, compared to the pre-crisis period. It is mainly due to fact that the GFC led to a general reassessment of risks understood in a very broad sense (the recent works on contagion, liquidity and counterparty risks constitute a convincing illustration).

Similar conclusions can be drawn for the CDS spread changes. The mean value of CDS changes is positive during the crisis period and the standard deviation is three times higher than the one obtained during the pre-crisis period, and twice as high as that observed during the post-crisis period. During this latter period the mean CDS changes are negative indicating a decline in the CDS spread and a posteriori justifying the name of post-crisis for the third sub-sample. As for

\(^{9}\)Although all companies are not mainly involved in the oil industry as we work at the index level we believe that this would not invalidate the results.

\(^{10}\)These are the days on which fortnightly averages are computed for oil futures.
the standard deviation, it is larger for the post-crisis period compared to the pre-crisis period, which also indicates a permanent increase in uncertainty in the credit market.

Although our study focuses on the relationship between volatility of the futures price and CDS spread, it is of interest to consider statistical properties of futures log-returns. Table 1 also reports the mean and standard deviation for futures log-returns. During the pre-crisis period the mean of the futures log-returns is positive corresponding to 0.0099, or 25.7% per annum, and the standard deviation is 0.0547. During the crisis period the mean is, as expected, negative and the corresponding standard deviation is twice as large as during the pre-crisis period. For the post-crisis sample the mean is positive corresponding to 0.0023, or 6% per annum, and the standard deviation is 0.0484, which is slightly smaller than the pre-crisis value and is consistent with the observations made for the bipower variation values. Figure 4 shows the evolution of the CDS spread and futures for the entire sample. Many of the statistical properties described above can be deduced from this figure.

[ Insert Figure 4 here ]

We also consider the CDS spreads by rating class: AA, A, BBB, BB. More precisely, we group the CDS spreads used to compute the index by rating (also provided by Markit) and compute a CDS spread index value for each rating category using an equal weighting scheme (the average numbers of entities are 10.2, 8.4, 19.4 and 6.7 for the rating AA, A, BBB and BB, respectively). Descriptive statistics are presented in Table 2. Similarly to the statistics produced for the CDS index, we observe that the means and standard deviations of credit spreads increase substantially as credit quality deteriorates from AA to BB throughout the entire period. The AA rating class has the lowest mean and standard deviation of 27.66 and 6.65, respectively, during the pre-crisis period. Due to higher levels of uncertainty in the financial market system during the GFC, the means and standard deviations corresponding to this period increase. Both statistics increase when moving from the highest (AA) to the lowest (BB) rating class. The highest levels for the mean and standard deviation of 422.99 and 124.76, respectively, are achieved for the lowest (BB) rating class. As noted earlier for the CDS spread level, both statistics are lower during the post-crisis period but the values remained higher compared to those for the pre-crisis period across all rating classes, which can be explained by the change in risk perception after the GFC.

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11 Other rating classes are not considered due to unavailability of data for the entire sample period.
along with the overhaul of market regulations, and reinforcement of collateralization and funding aspects. We also present Figure 5 which compares CDS spreads for the different rating classes. We note that as the credit quality deteriorates, there is an increase in the CDS spread levels throughout the entire sample period. All rating classes evolve in a similar manner but we note that the CDS level has increased by a factor of approximately 2 for BBB and BB when moving from the pre-crisis to the crisis period, and by a factor more than 2 for AA and A, indicating that the higher rating categories have been affected more during the GFC, compared to the lower rating categories.

\begin{center}
\[ \text{[ Insert Table 2 here]} \]
\[ \text{[ Insert Figure 5 here]} \]
\end{center}

The mean values for CDS changes during the crisis period across all rating classes are positive which is consistent with the substantial increase of credit spread levels over that period. As expected, the lowest mean value during the crisis period corresponds to the AA rating class while the highest is attributed to the BB category. Similar results can be drawn for the standard deviations, which are higher during the crisis period across all rating classes with pre-crisis values being lower than those for post-crisis.

4 Methodology and Empirical Results

4.1 Regression analysis

The importance of jumps as an explanatory variable has been convincingly illustrated in Tauchen and Zhou (2011), Wright and Zhou (2009) and Zhang et al. (2009). Tauchen and Zhou (2011) show that the jump volatility (that is, the volatility of \( \hat{J}_t \) defined in Eq.(8)) explains a large portion of bond spreads. They use regression to analyze the relevance of jump volatility computed from a one-year or two-year rolling window on monthly AAA and BAA bond spreads\(^{12}\). They obtain large \( R^2 \), around 20% when performing simple regression; see Table 5 in Tauchen and Zhou (2011) for more details. However, this choice of a rolling window implies that volatility observations are computed on overlapping intervals, which implies strong autocorrelation of the

\(^{12}\)These ratings are provided by Moody’s and correspond to AAA and BBB of Markit.
explanatory variable and might be problematic if the sample size is small. Say, if the sample size is close to two years, a large part of the sample is used to compute volatility, and, therefore, only a few observations can be built leading to unreliable regression results. In addition, one should note that Tauchen and Zhou (2011) consider bond yield spread *levels* that assume stationarity of the yield. This assumption is clearly not satisfied during the GFC. Another interesting contribution is that of Wright and Zhou (2009) who illustrate the importance of the mean jump size of the 30-year Treasury bond futures to explain the monthly excess return on holding of an *n*-month maturity bond (with *n* = 2; 24; 36; 48; 60). The mean value is computed using a 24-month rolling window, thus imposing a constraint on the sample size when applying this methodology. In addition, the authors demonstrate that jump volatility and intensity are not significant (see Table 2 in Wright and Zhou (2009) for details). It remains unclear whether their conclusions remain valid for a rolling window of a smaller size, which is crucial if the underlying sample size is small. Of particular interest to us is a study by Zhang et al. (2009) who investigate the CDS market at a firm level. The authors show that jump activity has a strong explanatory power for corporate CDS spreads for a sample ranging from 2001 to 2003 at weekly frequency. Positive and negative jumps denoted as *J*^+^*_t_ and *J*^-^*_t_ , respectively, have significant coefficients and are able to explain CDS spread levels. Jump volatility, on the contrary, is not significant when considered jointly with continuous volatility given by the bipower realized volatility estimator *BV*_t_; see Table 3 in Zhang et al. (2009). Implementation strategies for both aforementioned papers require one-year (or two-year) averaging over the variables (either jumps or realized volatility) and can be problematic if the sample of interest is too small. The authors consider both, credit spread levels and credit spread changes but note that the results are less satisfactory when dealing with CDS spread changes. It is well known (Collin-Dufresne et al. (2001)) that spread changes are much more difficult to explain and generally result in a smaller *R*^2_. Consistent with this remark Zhang et al. (2009) found an *R*^2_ of around 4% and, interestingly, the jumps (either positive or negative, or their intensity) are all statistically insignificant (see regression 2 in Table 5 in Zhang et al. (2009)). Focusing on CDS spread changes instead can be necessary if the underlying time series exhibits a trend, and this situation is especially pronounced during the GFC.

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13 Specifically in Wright and Zhou (2009) this aspect is not an issue since their sample ranges from July 1982 to September 2006.

14 The problem of autocorrelation of explanatory variables in the computation of the t-statistics can be resolved by using Newey-West’s result.
The objective of this paper is to analyze the role of jumps extracted from the futures light sweet crude oil in explaining CDS spread changes for the energy market. We focus on CDS changes rather than levels because our sample contains the GFC and, therefore, is not stationary. The Augmented Dickey-Fuller (ADF) test with the null hypothesis $H_0$: the series contains unit root (i.e. non-stationary) has been applied to CDS data (at an index level and by rating group) as well as to BV, across various time periods. The associated p-values are reported in Table 3.

As it is evident from the results, p-values for all the CDS data (at an index level and by rating group, as well as across the entire sample and sub-samples) are above 0.05, thus, the null hypothesis of non-stationarity could not be rejected at 5% significance level. Thus, we conclude that the series are not stationary. The p-values for the $BV$ across the entire sample, the pre-crisis and the post-crisis periods are below 0.05, indicating the rejection of the null and thus, stationarity of the series. The only sub-sample for which $BV$ is not stationary is a sub-sample containing crisis, where the the p-value corresponds to 0.2521. To take the reported results into consideration and to make our regression analysis consistent across sub-samples and rating groups, we choose to work with series in first differences. Contrary to existing literature, we consider fortnightly observations where explanatory variables are averaged over two weeks. This procedure avoids overlapping in the averaging process and allows us to work with relatively small sample sizes. As a result, the averaging is significantly smaller than in existing studies. As an example, we are able to consider GFC which ranges from January 2008 to December 2009, comprising 50 fortnightly observations. To which extent we need to average jumps to measure their impact on a dependent variable is unclear as the literature provides no guidance on that aspect. There is also no indication on the size of averaging jumps’ size or jumps’ volatility and whether it should be related to the frequency of the dependent variable, but clearly the averaging is necessary in order to have an effect that is significant. Ultimately, the objective is to understand if two assets (i.e. CDS spread and futures in our case) jump together. To analyze that problem there is (at least) one framework, see Jacod and Todorov (2009). However, jump estimation requires a lot of data, typically high frequency data, that are not often available for some asset classes. As a result, we are led to measure the impact of jumps, measured for one asset class, on another asset class for which only a crude description is available (because
of lack of high frequency data). This forces us to average the jumps to obtain a significant effect.

From an analytical point of view, we consider the regression equation of the form

$$\Delta CDS_t = c_0 + c_1 \Delta BV_t + c_2 J_t^+ + c_3 J_t^- + \epsilon_t.$$  \hspace{1cm} (12)

We do not incorporate VIX, treasury yield curve or other explanatory variables since our purpose is to focus exclusively on the relationship between CDS spreads and futures volatility and jumps (see Hammoudeh et al. (2015) for a joint analysis without jumps of oil-related CDS markets along with other global financial market variables; and Arouria et al. (2014) for an analysis of CDS spreads for financial sectors using several variables, one of which being the WTI crude oil futures contracts). Notice that the OVX, which is the CBOE Crude Oil Volatility Index, constitutes a more natural market-wide volatility indicator than the VIX; see Chevallier and Sévi (2013) for an analysis of the importance of variance risk premia based on OVX for predicting WTI light sweet crude oil futures. It will allow us to specify characteristics that the dynamics of the stock, its volatility and the CDS spread should have in order to be consistent with the empirical properties. Ultimately, we could draw conclusions on the form of a system of stochastic differential equations for these variables and compare it with the models available in the literature. Consistent with Merton (1974), we would expect negative correlation between stock movements and credit risk. When the stock price increases due to a positive jump, probability of default decreases, CDS drops and therefore, one would expect $c_2 < 0$. On the other hand, if the stock price decreases due to a negative jump, probability of default increases, which leads to an increase in CDS and subsequently, $c_3 < 0$. Finally, with decreasing stock price (and thus, increasing CDS) volatility will increase due to a leverage effect, and thus, one would expect $c_1 > 0$.

4.2 Analysis at the index level

We perform regression analysis based on Eq.(12) first using the entire sample and then for each sub-sample. To filter out jumps we used $\alpha = 0.999$ in Eq.(8), following Zhang et al. (2009)\textsuperscript{15}. The results are reported in Table 4.\textsuperscript{16} Whenever we refer to the regression coefficient

\textsuperscript{15}The results have been checked for robustness for different levels of $\alpha$, and are available from the authors upon request.

\textsuperscript{16}We notice that when estimating the regressions with Newey-West heteroscedasticity and autocorrelation-consistent (HAC) standard errors and an optimally chosen lag, we observe results consistent with those reported
as “significant”, we mean significance at 5% level, and will specify otherwise. For the entire sample we observe $R^2$ corresponding to 10.9%, which is an encouraging result given that CDS spread changes are difficult to explain, in contrast to CDS spread levels. The coefficient for the continuous volatility part, given by the bipower realized volatility estimator $BV_t$, is significant and positive (t-statistic of 2.31), which is consistent with the result in Merton (1974). Positive jumps are significant and the coefficient is negative (t-statistic of -2.58). Negative jumps have a strong negative and significant coefficient (t-statistic of -4.15), which underpins the importance of this jump type. Both results are again in line with Merton (1974). Comparing positive and negative jump sizes along with the estimated coefficients we conclude that negative jumps, although occurring slightly less frequent, have a larger price impact on CDS spreads, compared to positive jumps. Summarizing, all the estimates are consistent with Merton (1974)’s work and negative jumps are an important ingredient in the dynamics of futures prices to explain CDS spreads\textsuperscript{17}. We now perform the regression on the different sub-samples to understand how market conditions affect the results.

[ Insert Table 4 here ]

During the pre-crisis period, ranging from January 2004 to December 2007, the regression analysis leads to insignificant coefficients for both $BV_t$ and $J_+^t$. Only negative jumps appear to be significant and have a correct (negative) sign for the regression coefficient (t-statistic of -2.10). Furthermore, the $R^2$ is 5.2% which is rather low but is in line with the results reported in Zhang et al. (2009). Note that this weak relationship between futures volatility and CDS spreads echoes the low correlation between futures log-returns and CDS spread changes reported in Table 5. Although the relationship between futures’ volatility and CDS spreads is weak during the pre-crisis period (implying loose market linkages), the signs of this relationship are consistent with Merton (1974)’s results.

[ Insert Table 5 here ]

in Table 4. The results are not reported here but are available from the authors upon request.

\textsuperscript{17}Note that we do not expect the results to be inconsistent with Merton’s intuition (even though Merton has not incorporated jumps into his model). In fact, any market will comply with Merton’s intuition, but nevertheless we will be able to draw conclusions that could potentially lead to some open problems.
For the crisis period the results are satisfactory as $R^2$ reaches 16.4%. The continuous part of the volatility, $BV_t$, is not significant although it has a correct (positive) sign. Both negative and positive jump coefficients have a correct (negative) sign, with negative jumps being significant (t-statistic of -2.23) and positive jumps being nearly significant (t-statistic of -1.93) suggesting that during financial turmoils stock price dynamics have more discontinuities, and information is impounded in the price abruptly. Indeed, from the descriptive statistics in Table 1 we note that jumps are substantially larger (in absolute value terms) during the crisis period and display larger variability. From Table 5 we observe that correlation between futures log-returns and CDS spread changes become strong and negative. To summarize, jump modeling is essential if the sample under consideration contains turbulent periods. In addition, the relationship between variables under consideration is stronger during such periods, which is natural in view of Merton’s model.

For the last sub-sample, the post-crisis period, the regression analysis also leads to interesting results. Firstly, $R^2$ is high at 14% which implies that volatility, understood in a broad sense (that is, continuous and discontinuous), is able to explain CDS spread changes, which contrasts with the results for the pre-crisis period where $R^2$ only reaches 5%. Higher $R^2$ during the post-crisis period compared to pre-crisis also implies that the relationship between the CDS market and the volatility market has been reinforced after the GFC; that is, the markets became more connected. This aspect can be further confirmed when looking at the correlation level between futures log-returns and CDS spread changes that is nearly twice the pre-crisis value; see Table 5. In this particular case the significant variables are the bipower realized volatility (t-statistic of 2.73) and the negative jumps (t-statistic of -2.15), both with correct signs. The results provide useful guidance to build a parametric model.

To illustrate the results described above in a time-varying setting, we plot time-varying t-test statistics estimated using a moving window of 78 fortnightly observations (corresponding to approximately 3 years of data) in Figure 6. The first test statistic is computed using 78 fortnightly observations from Wednesday, 21 January 2004 to Wednesday, 15 January 2005; the second test statistic is computed using observations from Wednesday, 4 February 2005 to Wednesday, 29 January 2006, etc. The results are consistent with those observed in Table 4; $R^2$ ranges from 5% for the pre-crisis period to above 20% during the crisis period. $BV_t$ is mostly insignificant but has a correct (positive) sign. Positive jumps are mostly negative (with an exception of the
post-crisis period) and significant during the crisis period. Negative jumps are negative and significant, with significance increasing during the crisis period.

[ Insert Figure 6 here ]

Putting the results obtained for the three sub-samples in perspective with those for the entire sample suggests a strengthening of the relationship between the credit and volatility markets that has certainly been triggered by the GFC.

So far we have worked at an index level which averages the CDS spreads for all U.S. energy companies. It is instructive to disaggregate the data and to perform this analysis for different rating groups that constitute the index to understand how creditworthiness affects the relation between CDS spread changes and futures volatility.

4.3 Analysis by rating group

We report in Table 6 the regression results for different samples and rating groups (AA, A, BBB, BB).

For the entire sample, across all rating groups all coefficients have correct signs. Namely, positive sign is observed for the bipower realized volatility and negative for both jump components. We also note the discrepancies between the coefficients across different rating groups that suggest a dependency of the CDS-volatility relationship on creditworthiness. This implies that a panel analysis, which imposes the same relation across rating groups, is likely to perform poorly, and might explain the low $R^2$ in regression 2, Table 5 of Zhang et al. (2009).

Negative jump activity is extremely important for all rating groups as the coefficients are all significant (ranging from -2.18 for AA to -3.78 for BBB). For the positive jumps, they are significant for all but the A rating. Rather surprising is the result for the bipower realized volatility that is significant for the A rating only. We now focus on regression analysis for the different sub-samples but we first raise some remarks on their validity. Whenever the coefficient is significant, its sign is consistent with Merton (1974)’s model. For a given sub-sample the

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18 We notice that when estimating the regressions with Newey-West heteroscedascity and autocorrelation-consistent (HAC) standard errors and an optimally chosen lag, we observe results consistent with those reported in Table 6. The results are not reported here but are available from the authors upon request.

19 T-statistic for BBB rating is nearly significant taking the value of -1.93. Note that if we assume lower significance level, our results will become significant for all rating classes, including A.
regression coefficients across all rating groups vary considerably. Similarly to the results reported for the entire sample, we observe discrepancies between the coefficients across different rating groups, suggesting that panel analysis might be inappropriate even for the sub-sample periods.

Considering the pre-crisis period, negative jumps are significant only for the A rating class whereas positive jumps are significant only for the AA rating class. These two groups also have the highest $R^2$ corresponding to 6.9% and 10.8%, respectively. However, the overall relationship between volatility and CDS spread changes appears to be weak.

For the crisis sample period, the A, BBB and BB rating classes display high $R^2$ values ranging from 13.4% to 17.8%. Negative jumps appear to be significant for lower rating categories (i.e., BBB and BB) whereas positive jumps are significant for the lowest rating BB only. This result is to be expected since when rating quality deteriorates, negative jumps will have a stronger impact on the probability of default. For the A rating class, connection between CDS spread changes and realized volatility is explained through the bipower realized variation $BV_t$. Overall, jumps are important for most of the categories. Interestingly, for the highest rating group AA none of the coefficients are significant and $R^2$ is extremely low (3.4%). This latter result suggests a loose relationship between the high rated CDS market and the futures market: this CDS rating category might have been considered as out of trouble by market participants irrespective of the events occurring in the financial markets or, stated differently, this rating category was the safest available on the market and, as a result, the least to require a cross-hedging activity. During the post-crisis period negative jumps are significant for all but the lowest rating group (BB) while the bipower realized volatility is significant for the BB rating class only. $R^2$ is slightly higher for the post-crisis period compared to the pre-crisis period.

When comparing correlations reported in Table 5 between futures log-returns and CDS spread changes by rating class and across sub-samples, the results appear consistent with those observed for the entire sample: the lowest correlations (in absolute terms) are observed for the pre-crisis period for all rating groups, with a subsequent increase during the crisis period and a minor drop afterwards during the post-crisis period. We note that correlations become stronger after the crisis compared to pre-crisis for all rating groups except for the highest rating AA.

Overall, our results underline the importance of negative jumps and the bipower realized volatil-

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20This latter result seems consistent with the “flight to quality” effect observed during the crisis.
ity (i.e., the continuous part of the volatility) that are required for accurate modelling of the relationship between the CDS market and the volatility market. This relationship depends on global market conditions and appears to be strong during a bear market configuration. It explains the vivid interest among academics for works related to contagion effects across markets. Our conclusions are valid at an index level as well as per rating group.

5 Consistent Model Perspective

From our results we can derive hedging strategies between the CDS market and the stock market, but we can also devise the specifications of a joint model for these two markets. It is well known that consistent pricing of CDS spreads suggests that the dynamics of the default intensity is given by a diffusion process of an affine type; see Duffie and Singleton (1999). It is also well known that in the first order approximation CDS spread can be approximated by credit default intensity. For the stock dynamics the affine framework, with or without jumps, is by far the most commonly adopted framework; see for example Duffie et al. (2000). Therefore, in order to consistently price CDS spreads and stock option products the affine framework is a natural choice. Our results underlie the importance of jumps for a realistic modeling of the interaction between the stock and the CDS market. As such, a joint model should have jumps affecting both, the stock and the default intensity. This feature seems to be even more important than the relationship between the stock volatility and the default intensity.

Considering the models available in the literature so far it seems that none of them satisfy these empirical constraints. The existing models have been already discussed in the introduction, but let us make our remarks more precise, in order to put our results into a model specification perspective and illustrate the fact that they have far reaching consequences. For example, in Carr and Wu (2007) jumps on the stock do not affect the default intensity. In addition, the link between the stock volatility and the default intensity is carried out through the parameter $\beta$ (in Eq. (6) of Carr and Wu (2007)) that is found, quite surprisingly, to be negative, see column I, Table 5 for both countries analyzed in this paper. This result is rather problematic as a positive sign is expected due to the fact that a negative value would imply a negative default intensity, and a negative correlation between the stock volatility and the default intensity. The
link between the CDS default intensity and the stock dynamics is performed through the stock jump intensity (i.e. the jump arrival rate) and not the jump size, contrary to the empirical facts enlighten by our results. A similar model was implemented in Carr and Wu (2010) using another data set and the results are also slightly disappointing. The parameter $\beta$, which links the CDS default intensity and the stock price dynamics, is found to be not significantly different from zero (see Table 6), thus suggesting a zero correlation between the default intensity and the stock volatility. This model is different from Carr and Wu (2007) in a subtle but important way. As the stock jump intensity is driven by the stock volatility, the stock/volatility dynamics are, therefore, completely unrelated to the CDS intensity. In other words, the CDS market is independent of the stock/volatility market. There are at least two possible explanations for this result: either for the markets studied in these papers there is no interaction between the CDS spread and the stock dynamic or, which appears to be most likely, the considered model lacks flexibility to cope with this interaction. In any case, for the energy market these models do not possess the relevant properties. Our results unambiguously point towards the importance of jumps in the relationship between the CDS and the stock/volatility markets. In conclusion, our work suggests to investigate the joint dynamics between these markets using the following specification for stochastic volatility model:

$$\begin{align*}
\frac{dS_t}{S_t} &= \mu_t dt + \sqrt{v_t} dw^1_t - (e^{J^s} - 1) dN_t \\
v_t &= \kappa(\theta - v_t) dt + \sigma \sqrt{v_t} dw^2_t \\
z_t &= \kappa_z(\theta_z - z_t) dt + \sigma_z \sqrt{z_t} dw^3_t \\
\lambda_t &= v_t + z_t + J^\lambda dN_t
\end{align*}$$

with $w_t = (w^1_t, w^2_t, w^3_t)$ being a three-dimensional Brownian motion with correlation structure $d\langle w^1, w^2 \rangle_t = \rho dt$, $d\langle w^1, w^3 \rangle_t = d\langle w^2, w^3 \rangle_t = 0 dt$, $\{N_t; t \geq 0\}$ a Poisson process with constant arrival rate, $J^s$ and $J^\lambda$ the jump sizes for the stock and the CDS default intensity given by $\lambda_t$, $v_t$ is the stock volatility and $z_t$ is an additional factor. Notice that for the above specification the linkage between the stock/volatility market and the credit risk market is not performed through the jump intensity. Our simple framework allows us to confirm the weak dependency between the CDS spread changes and the stock jump intensity.

We compute fortnightly using Eq.(11) the positive and negative jump intensities and perform
the following simple regressions

$$\Delta \text{CDS}_t = c_0^j + c_1^j \Delta \lambda^j_t + \epsilon^j_t \quad j \in \{+,-\}. \quad (13)$$

We expect $c_1^+$ to be negative while $c_1^-$ should be positive. The results are reported in Table 7 at the index level and in Table 8 per rating group. As for the previous cases the analysis is performed for the entire sample and for each sub-sample. At the index level for the entire sample as well as for each sub-sample both, $\lambda^+_t$ and $\lambda^-_t$ are insignificant, indicating these quantities cannot explain the CDS spread changes. These results are in a sharp contrast with the explanatory power of jump sizes, as reported in Table 4.

[ Insert Table 7 here ]

If we consider the results per rating group the conclusions barely change. Table 8 shows that positive jump intensity $\lambda^+_t$ is significant only for the rating group AA during the pre-crisis period (with the correct sign). Regarding the negative jump intensity (i.e. $\lambda^-_t$), it is not significant, for any of the rating classes or any sub-sample. Again, this sharply contrasts with the results obtained in Table 6. It appears to us that the conclusion is unambiguous, the use of jump intensity is not the best way to establish a link between the CDS market and the equity/volatility market, to perform market linkages. This may explain the poor empirical findings obtained so far and suggests that other model specifications are required.

[ Insert Table 8 here ]

6 Conclusion

This paper presents a comprehensive study on the relationship between CDS spreads for the Energy sector and oil futures dynamics. Motivated by Zhang et al. (2009) who examine to what extent equity jumps and volatility are determinants of CDS spreads, we analyze the importance of volatility and jumps extracted from light sweet crude oil futures in explaining CDS spread changes for the energy market. Our sample is large, ranging from January 2004 to December 2013, and covers the Global Financial Crisis, hence the fundamental relationship between credit
risk and futures volatility and jumps can be analyzed under very different market conditions. The analysis is performed at an index level and per rating group thus assessing the impact of creditworthiness on this relationship, as well as for different sub-samples (pre-crisis, crisis and post-crisis periods).

Our findings are consistent with Merton’s theoretical framework. First of all, at an index level, futures jumps are important when explaining CDS spread changes, with negative jump components having higher impact during the crisis period. Furthermore, the continuous volatility part, given by the bipower realized volatility, is significant and positive indicating that futures volatility conveys relevant information for the CDS market. In terms of the results per rating group, we find that negative jumps have an increasing importance as the credit rating deteriorates while futures volatility becomes more important for higher rating categories. For the highest rating category the CDS spread depends very weakly on both futures jumps and volatility. Finally, the relation between the CDS market and the futures market appears to be stronger during volatile periods and strengthened after the Global Financial Crisis.

Our work suggests several extensions. First, our analysis has been performed for the CDS U.S. market, a worldwide extension is definitively of interest. Second, we restrict our study to the pair CDS for the energy sector, constituting mainly of oil companies, and oil futures, but other commodities and/or other countries are worth considering. Metal, gas, agriculture and electricity futures jointly analyzed with CDS spreads for companies depending on these commodities could lead to a general overview of the credit risk and futures relationship for commodity markets. Regarding the electricity market, it is well known that jumps (spikes) play a very particular role; it would be of interest to understand how this affects the results. Third, in the present work the realized volatility and jump activities are examined under the historical probability. Alternatively, as mentioned in the introduction, one could extract volatility through options, or more generally through derivatives, which will result in dynamics under the risk neutral probability measure. These two dynamics, and more precisely their difference, that is, the variance risk premium that was extensively studied for commodity markets in Prokopczuk and Wese Simen (2014), is known to be of crucial importance and quantifying its impact on the credit default swap market is essential. Furthermore, our results suggest that the dynamics
of the stock with jumps have an impact on the CDS spread, which differs significantly from the consistent models proposed in Carr and Wu (2007, 2010). As a result, performing a joint calibration of stock options and CDS spread term structure for the Energy sector with a model having the specifications underlined by our work constitutes an open and promising objective. Lastly, our analysis is performed at an index level as well as per rating group, which is a first stage of index disaggregation. Working at a firm level will provide a complete picture of the fundamental relation found by Merton. It appears to us that all these important questions that extend naturally our results have not yet been considered in the literature; they are left for future research.
References


A Appendix

A.1 Tables

<table>
<thead>
<tr>
<th>Table 1: Descriptive Statistics</th>
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<tr>
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<tr>
<td>Mean CDS_t level</td>
</tr>
<tr>
<td>Std. dev. CDS_t level</td>
</tr>
<tr>
<td>Mean ΔCDS_t</td>
</tr>
<tr>
<td>Std. dev. ΔCDS_t</td>
</tr>
<tr>
<td>Futures log-returns</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Std. dev.</td>
</tr>
<tr>
<td>Mean BV_t level</td>
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<tr>
<td>Mean ΔBV_t</td>
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<td>Std. dev.</td>
</tr>
<tr>
<td>Mean J^+_t</td>
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<tr>
<td>Std. dev.</td>
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<tr>
<td>Mean J^-_t</td>
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<tr>
<td>Std. dev.</td>
</tr>
<tr>
<td>Mean λ^+</td>
</tr>
<tr>
<td>Mean λ^-</td>
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</table>

Note. Descriptive statistics computed using fortnightly observations from the entire sample (January 2004 to December 2013), pre-crisis (January 2004 to December 2007), crisis (January 2008 to December 2009) and post-crisis (January 2010 to December 2013) periods for CDS level and changes, futures log-returns, bipower variation (BV_t) level and changes, positive and negative jumps (J^+_t and J^-_t). Jump intensities are denoted by λ^+ and λ^- for positive and negative jumps, respectively. Jumps are filtered out using equation Eq.(8) with α = 0.999.
Table 2: Descriptive Statistics for CDS by Rating Group

<table>
<thead>
<tr>
<th>Rating Group</th>
<th>Entire period</th>
<th>Pre-crisis</th>
<th>Crisis</th>
<th>Post-crisis</th>
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<tr>
<td></td>
<td>CDS t level</td>
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<td></td>
<td></td>
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<td>51.27</td>
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<tr>
<td>BBB</td>
<td>Mean</td>
<td>122.26</td>
<td>81.86</td>
<td>194.28</td>
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<td></td>
<td>Std. dev.</td>
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<td>12.20</td>
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<td>Mean</td>
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<td>55.10</td>
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<td>ΔCDS t</td>
<td>AA</td>
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<td>-0.0466</td>
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<td>22.38</td>
<td>49.61</td>
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</tbody>
</table>

Note. Descriptive statistics computed using fortnightly observations from the entire sample (January 2004 to December 2013), pre-crisis (January 2004 to December 2007), crisis (January 2008 to December 2009) and post-crisis (January 2010 to December 2013) periods for CDS level and changes by rating group.

Table 3: Stationarity Tests Results

<table>
<thead>
<tr>
<th>Rating Group</th>
<th>Entire period</th>
<th>Pre-crisis</th>
<th>Crisis</th>
<th>Post-crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDS Index</td>
<td>0.3348</td>
<td>0.5200</td>
<td>0.6248</td>
<td>0.2743</td>
</tr>
<tr>
<td>CDS AA</td>
<td>0.2275</td>
<td>0.4202</td>
<td>0.4812</td>
<td>0.3302</td>
</tr>
<tr>
<td>CDS A</td>
<td>0.2702</td>
<td>0.3847</td>
<td>0.5457</td>
<td>0.3629</td>
</tr>
<tr>
<td>CDS BBB</td>
<td>0.3715</td>
<td>0.5574</td>
<td>0.5762</td>
<td>0.3777</td>
</tr>
<tr>
<td>CDS BB</td>
<td>0.2633</td>
<td>0.3973</td>
<td>0.6093</td>
<td>0.2315</td>
</tr>
<tr>
<td>BV</td>
<td>0.0054</td>
<td>0.0379</td>
<td>0.2521</td>
<td>0.0084</td>
</tr>
</tbody>
</table>

Note. P-values obtained from the Augmented Dickey-Fuller (ADF) test with the null hypothesis $H_0$ : the series contains unit root (i.e. non-stationary). Values above 0.05 indicate rejection of the null at 5% significance level. The test is performed using fortnightly observations CDS (at an index level and by rating group) as well as for BV, across various time periods.
Table 4: Regression Analysis

<table>
<thead>
<tr>
<th>Entire period: Jan 2004 - Dec 2013</th>
<th>( \Delta B V_t )</th>
<th>( J^+_t )</th>
<th>( J^-_t )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.218 \times 10^4</td>
<td>-0.102 \times 10^4</td>
<td>-0.139 \times 10^4</td>
<td>0.109</td>
<td></td>
</tr>
<tr>
<td>(2.31)</td>
<td>(-2.58)</td>
<td>(-4.15)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-crisis: Jan 2004 - Dec 2007</td>
<td>( 6.810 \times 10^3 )</td>
<td>-0.224 \times 10^3</td>
<td>-1.072 \times 10^3</td>
<td>0.052</td>
</tr>
<tr>
<td>(0.97)</td>
<td>(-0.47)</td>
<td>(-2.10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crisis: Jan 2008 - Dec 2009</td>
<td>( 1.114 \times 10^4 )</td>
<td>-0.200 \times 10^4</td>
<td>0.180 \times 10^4</td>
<td>0.164</td>
</tr>
<tr>
<td>(0.91)</td>
<td>(-1.93)</td>
<td>(-2.23)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post-crisis: Jan 2010 - Dec 2013</td>
<td>( 3.024 \times 10^4 )</td>
<td>0.054 \times 10^4</td>
<td>-0.143 \times 10^4</td>
<td>0.140</td>
</tr>
<tr>
<td>(2.73)</td>
<td>(0.75)</td>
<td>(-2.15)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note.* Regression results for the equation \( \Delta \text{CDS}_t = c_0 + c_1 \Delta BV_t + c_2 J^+_t + c_3 J^-_t + \epsilon_t \) obtained using fortnightly observations from the entire sample (January 2004 to December 2013), pre-crisis (January 2004 to December 2007), crisis (January 2008 to December 2009) and post-crisis (January 2010 to December 2013) periods. Jumps are filtered out using equation Eq.(8) with \( \alpha = 0.999 \). For a given sample and rating we report the coefficient estimates and corresponding t-statistics in parenthesis (see also footnote 16).

Table 5: Futures and CDS Correlations

<table>
<thead>
<tr>
<th></th>
<th>Entire period</th>
<th>Pre-crisis</th>
<th>Crisis</th>
<th>Post-crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index</td>
<td>-0.2639</td>
<td>-0.1411</td>
<td>-0.3341</td>
<td>-0.2207</td>
</tr>
<tr>
<td>AA</td>
<td>-0.1524</td>
<td>-0.0515</td>
<td>-0.1682</td>
<td>-0.1908</td>
</tr>
<tr>
<td>A</td>
<td>-0.2049</td>
<td>-0.0967</td>
<td>-0.2407</td>
<td>-0.2290</td>
</tr>
<tr>
<td>BBB</td>
<td>-0.3158</td>
<td>-0.1844</td>
<td>-0.3970</td>
<td>-0.2163</td>
</tr>
<tr>
<td>BB</td>
<td>-0.2313</td>
<td>-0.1129</td>
<td>-0.2888</td>
<td>-0.2046</td>
</tr>
</tbody>
</table>

*Note.* Correlations between fortnightly futures log-returns and CDS differences for the entire sample (January 2004 to December 2013), pre-crisis (January 2004 to December 2007), crisis (January 2008 to December 2009) and post-crisis (January 2010 to December 2013) periods.
Table 6: Regression Analysis by Rating Group

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta BV_t$</td>
<td>$J_t^+$</td>
<td>$J_t^-$</td>
<td>$R^2$</td>
</tr>
<tr>
<td>AA</td>
<td>$8.045 \times 10^2$</td>
<td>$-3.664 \times 10^2$</td>
<td>$-3.470 \times 10^2$</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>(0.32)</td>
<td>(-1.96)</td>
<td>(-2.18)</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>$1.116 \times 10^4$</td>
<td>$-0.056 \times 10^4$</td>
<td>$-0.0869 \times 10^4$</td>
<td>0.096</td>
</tr>
<tr>
<td></td>
<td>(2.77)</td>
<td>(-1.87)</td>
<td>(-3.40)</td>
<td></td>
</tr>
<tr>
<td>BBB</td>
<td>$5.313 \times 10^4$</td>
<td>$-0.723 \times 10^3$</td>
<td>$-1.200 \times 10^3$</td>
<td>0.071</td>
</tr>
<tr>
<td></td>
<td>(1.06)</td>
<td>(-1.93)</td>
<td>(-3.78)</td>
<td></td>
</tr>
<tr>
<td>BB</td>
<td>$1.478 \times 10^4$</td>
<td>$-0.244 \times 10^4$</td>
<td>$-0.216 \times 10^4$</td>
<td>0.061</td>
</tr>
<tr>
<td></td>
<td>(1.29)</td>
<td>(-2.86)</td>
<td>(-2.98)</td>
<td></td>
</tr>
</tbody>
</table>

Note. Regression results (per rating group) for the equation $\Delta CDS_t = c_0 + c_1 \Delta BV_t + c_2 J_t^+ + c_3 J_t^- + \epsilon_t$ obtained using fortnightly observations from the entire sample (January 2004 to December 2013), pre-crisis (January 2004 to December 2007), crisis (January 2008 to December 2009) and post-crisis (January 2010 to December 2013) periods. Jumps are filtered out using equation Eq.(8) with $\alpha = 0.999$. For a given sample and rating we report the coefficient estimates and corresponding t-statistics in parenthesis (see also footnote 18).
Table 7: Regression on Intensity

<table>
<thead>
<tr>
<th></th>
<th>$\Delta \lambda^+$</th>
<th>$R^2$</th>
<th>$\Delta \lambda^-$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Entire period: Jan 2004 - Dec 2013</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-1.700</td>
<td>0.000</td>
<td>6.398</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>(-0.32)</td>
<td>(1.29)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Pre-crisis: Jan 2004 - Dec 2007</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.334</td>
<td>0.001</td>
<td>7.387</td>
<td>0.018</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
<td>(1.34)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Crisis: Jan 2008 - Dec 2009</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-12.648</td>
<td>0.009</td>
<td>19.474</td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td>(-0.67)</td>
<td>(1.03)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Post-crisis: Jan 2010 - Dec 2013</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.928</td>
<td>0.001</td>
<td>0.984</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td>(0.17)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. Regression results for the simple regression model in equation $\Delta \text{CDSS}_t = c_0^j c_1^j \Delta \lambda^j_t + \epsilon^j_t$  $j \in \{+, -\}$ obtained using fortnightly observations from the entire sample (January 2004 to December 2013), pre-crisis (January 2004 to December 2007), crisis (January 2008 to December 2009) and post-crisis (January 2010 to December 2013) periods. Jumps are filtered out using equation Eq.(8) with $\alpha = 0.999$. 
Table 8: Regression on Intensity by Rating

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta \lambda^+$</td>
<td>$R^2$</td>
<td>$\Delta \lambda^-$</td>
<td>$R^2$</td>
<td>$\Delta \lambda^+$</td>
<td>$R^2$</td>
<td>$\Delta \lambda^-$</td>
<td>$R^2$</td>
</tr>
<tr>
<td>AA</td>
<td>-2.541 0.005</td>
<td>0.839 0.001</td>
<td></td>
<td>(-1.07) (0.37)</td>
<td></td>
<td>(-1.07) (0.37)</td>
<td></td>
<td>(-1.07) (0.37)</td>
</tr>
<tr>
<td>A</td>
<td>-0.634 0.000</td>
<td>6.884 0.013</td>
<td></td>
<td>(-0.16) (1.84)</td>
<td></td>
<td>(0.16) (-1.84)</td>
<td></td>
<td>(0.16) (-1.84)</td>
</tr>
<tr>
<td>BBB</td>
<td>-2.434 0.001</td>
<td>4.068 0.003</td>
<td></td>
<td>(-0.50) (0.88)</td>
<td></td>
<td>(0.50) (-0.88)</td>
<td></td>
<td>(0.50) (-0.88)</td>
</tr>
<tr>
<td>BB</td>
<td>-4.398 0.001</td>
<td>7.726 0.002</td>
<td></td>
<td>(-0.40) (0.74)</td>
<td></td>
<td>(0.40) (-0.74)</td>
<td></td>
<td>(0.40) (-0.74)</td>
</tr>
<tr>
<td>AA</td>
<td>-3.601 0.057</td>
<td>-1.605 0.010</td>
<td></td>
<td>(-2.46) (-1.01)</td>
<td></td>
<td>(-2.46) (-1.01)</td>
<td></td>
<td>(-2.46) (-1.01)</td>
</tr>
<tr>
<td>A</td>
<td>-3.879 0.010</td>
<td>5.055 0.016</td>
<td></td>
<td>(-1.02) (1.26)</td>
<td></td>
<td>(-1.02) (1.26)</td>
<td></td>
<td>(-1.02) (1.26)</td>
</tr>
<tr>
<td>BBB</td>
<td>0.549 0.000</td>
<td>4.024 0.005</td>
<td></td>
<td>(0.10) (0.68)</td>
<td></td>
<td>(0.10) (0.68)</td>
<td></td>
<td>(0.10) (0.68)</td>
</tr>
<tr>
<td>BB</td>
<td>8.875 0.004</td>
<td>17.564 0.014</td>
<td></td>
<td>(0.62) (1.17)</td>
<td></td>
<td>(0.62) (1.17)</td>
<td></td>
<td>(0.62) (1.17)</td>
</tr>
</tbody>
</table>

Note. Regression results for the equation $\Delta \text{CDS}_t = c_0^j + c_1^j \Delta \lambda_t^j + \epsilon_t^j$ \( j \in \{+,-\} \) by rating group obtained using fortnightly observations from the entire sample (January 2004 to December 2013), pre-crisis (January 2004 to December 2007), crisis (January 2008 to December 2009) and post-crisis (January 2010 to December 2013) periods. Jumps are filtered out using equation Eq.(8) with $\alpha = 0.999$.  

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A.2 Figures

Figure 1: Annualized daily volatility $\sqrt{250} \times RV_t$ for the period from January 2004 to December 2013.

Figure 2: Bipower variation $BV_t$ (daily observations) for the period from January 2004 to December 2013.
Figure 3: Jump components $J_t$ (daily observations) for the period from January 2004 to December 2013.

Figure 4: Energy sector CDS (5-year maturity) and front futures, daily data from January 2004 to December 2013.
Figure 5: Energy sector CDS spread levels for different rating classes, daily data from January 2004 to December 2013.
Figure 6: Time-varying t-statistics computed from the regression equation $\Delta \text{CDS}_t = c_0 + c_1 \Delta BV_t + c_2 J_t^+ + c_3 J_t^- + \epsilon_t$ using fortnightly observations from January 2004 to December 2013. Moving window of 78 fortnightly observations (3 years) is used. Jumps are filtered out using equation Eq.(8) with $\alpha = 0.999$. 