An Analytical Method for Centroid Computing and Its Application in Wireless Localization

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12 December 2013
Outline

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Background

• Wireless Sensor Networks
  – Wide application scenarios
  – Omni-directional transmission gives a coverage roughly like a circle
  – Scarce power resources, usually operating on batteries

• Wireless localization Technique
  – Location information is necessary for meaningful data collection.
  – Localization Techniques
    • Region-based
    • Range-based
Centroid Computing

- **Centroid**
  - the point at which a cardboard cut-out of the region could be perfectly balanced on the tip of a pencil, with the assumption of uniform density and uniform gravitational field.

- **How to locate the centroid?**
  - Plumb line method
  - Geometric decomposition method
  - By integral formula

It is not difficult to compute the centroid of a half circle. But how about the centroid for the overlapped region of two intersecting circles (see the figure on the left)?
Proposed Analytical Method

- Overlapped area of two intersecting circles

\[
\begin{align*}
(x - x_1)^2 + (y - y_1)^2 &= r_1^2, \\
(x - x_2)^2 + (y - y_2)^2 &= r_2^2 \\
(x_s - x_1)^2 + (y_s - y_1)^2 &\leq r_1^2, \\
(x_s - x_2)^2 + (x_s - y_2)^2 &\leq r_1^2 \\
|r_1 - r_2| &\leq \sqrt{(x - x_1)^2 + (y - y_1)^2} \leq r_1 + r_2
\end{align*}
\]

- By integration

\[
(\bar{x}, \bar{y}) = \left( \frac{\iint_R x \, dx \, dy}{\iint_R dx \, dy}, \frac{\iint_R y \, dx \, dy}{\iint_R dx \, dy} \right)
\]
Analytical Solution for Special Case

\[ f(x_2) = \pm \sqrt{r_2^2 - (x - x_2)^2}, \text{ for left boundary} \]

\[ f(x_1) = \pm \sqrt{r_1^2 - (x - x_1)^2}, \text{ for right boundary} \]

\[ x = \frac{\int_{x_2 - r_2}^{0} 2x\sqrt{r_2^2 - (x - x_2)^2} \, dx + \int_{0}^{x_1 + r_1} 2x\sqrt{r_1^2 - (x - x_1)^2} \, dx}{\int_{x_2 - r_2}^{0} 2\sqrt{r_2^2 - (x - x_2)^2} \, dx + \int_{0}^{x_1 + r_1} 2\sqrt{r_1^2 - (x - x_1)^2} \, dx} \]
Analytical Solution for Special Case

\[ x = \frac{P}{Q} \]

\[ P = \frac{\pi}{4} \left( x_1 r_1^2 + x_2 r_2^2 \right) + \left( \frac{r_1^2}{3} + \frac{x_1^2}{6} \right) \sqrt{r_1^2 - x_1^2} \]
\[ - \left( \frac{r_2^2}{3} + \frac{x_2^2}{6} \right) \sqrt{r_2^2 - x_2^2} + \frac{r_1^2 x_1}{2} \sin^{-1} \left( \frac{x_1}{r_1} \right) - \frac{r_2^2 x_2}{2} \sin^{-1} \left( \frac{x_2}{r_2} \right) \]

\[ Q = \frac{\pi}{4} \left( r_1^2 + r_2^2 \right) + \frac{x_1}{2} \sqrt{r_1^2 - x_1^2} - \frac{x_2}{2} \sqrt{r_2^2 - x_2^2} \]
\[ + \frac{r_1^2}{2} \sin^{-1} \left( \frac{x_1}{r_1} \right) - \frac{r_2^2}{2} \sin^{-1} \left( \frac{x_2}{r_2} \right) \]
General Case

For the general case where the centres of two intersecting circles are not on the same axis, we need to:

1) Perform axis rotation
2) Perform axis shifting
3) Compute the centroid using the solution from special case
4) Perform axis shifting
5) Perform axis rotation and then obtain the analytical solution
Application of Analytical Centroid Computing

- Centroid-Based Localization (CBL)
  - Anchors deployment
    - e.g., A₁, A₂, A₃
  - Beacon frames broadcast
  - Beacon frames synthesis
    - Compute C₁,₂, C₁,₃, C₂,₃
  - Location estimation
    - Estimate sensor location for S

\[
S' : (x'_s, y'_s) = \left( \frac{x_{C_1,2} + x_{C_1,3} + x_{C_2,3}}{3}, \frac{y_{C_1,2} + y_{C_1,3} + y_{C_2,3}}{3} \right)
\]
Region-Based Localization

- To facilitate comparison, existing Region-Based Localization (RBL) was revisited here.
Worst-case Performance Comparison

![Graph showing performance comparison between different types of transmission ranges.](image)
Average Accuracy Comparison

![Graph showing accuracy comparison between CBL and result from [15].](image)
Effect of Mismatch in Anchor Transmission Power Levels
Conclusion

• An analytical approach of computing the centroid for the overlapped area of any two intersecting circles was presented.

• Centroid Based Localization (CBL) was proposed and its performance was compared with existing region based localization technique through computer simulations.

• CBL is able to improve localization accuracy while reducing transmission power of anchor nodes.

• The effect of power mismatch was also studied.
Q&A

Thank you for your attention!

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