Flexural motion of a semi-infinite floating plate under localised edge loading

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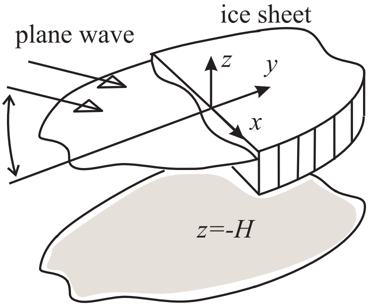


Semi-infinite elastic plate covers the half of the surface

Free-edge conditions

Reflection and transmission of an incident plane wave

$$\mathcal{I}e^{\mathrm{i}(x\kappa_0 \sin\theta + \kappa_0 y)} \longrightarrow \begin{cases} \mathcal{R}e^{\mathrm{i}(x\kappa_0 \sin\theta - \kappa_0 y)} & \text{for free-surface} \\ \\ \mathcal{T}e^{\mathrm{i}(x\kappa_0 \sin\theta + \mu_0 y)} & \text{for plate} \end{cases}$$

κ_0 : plate travelling wavenumber

μ_0 : free surface travelling wavenumber

Linear Wave-plate interaction:
Displacement $w(x, y, t)$, velocity potential $\phi(x, y, z, t)$

$$\begin{aligned} \nabla^4 w + \partial_{tt} w &= p(x, y, t) \text{ for plate} \\ \nabla^2 \phi &= 0 \text{ for body of water} \\ \phi_{tt} + \phi_z &= 0 \text{ for free surface,} \\ \rho\phi_t + \rho g w &= p(x, y, t) \text{ for plate,} \end{aligned}$$

Solutions by discrete modes

$$\phi(x, y, z) = \begin{cases} \sum a_n e^{\mathrm{i}(kx+\kappa_n y)} \cosh \kappa_n (z + H) & \text{for free-surface} \\ \sum b_n e^{\mathrm{i}(kx+\mu_n y)} \cosh \mu_n (z + H) & \text{for plate} \end{cases}$$

Some simple solutions

Non-dimesionalisation

$$l_c = \left(\frac{D}{\rho g}\right)^{1/4}, \quad time = \sqrt{\frac{l_c}{g}}$$

Non-dimensional mass density

$$\frac{m}{\rho l_c} \approx 0$$

Some simple solutions

Tkacheva (2001) derived the reflection coefficient for normal incident waves

$$\mathcal{R} = \left| \frac{\kappa_0 - \mu_0}{\kappa_0 + \mu_0} \right|$$

where

$$\kappa_0 \; : \; \text{plate travelling wavenumber}$$

$$\mu_0 \; : \; \text{free surface travelling wavenumber}$$

Solution for a given plane incident at an angle θ

$$\mathcal{I}(x) \sim e^{\mathrm{i}x\lambda_0 \sin\theta}$$

The solution

$$w(x,y) \sim w(x,\theta)e^{\mathrm{i}x\lambda_0 \sin\theta}$$

where λ_0 is the travelling wavenumber of free surface for a given time-frequency ω

The velocity potential

$$\phi\left(x,y,z\right)=d_0c_0\left(x,y,z\right)+d_1c_1\left(x,y,z\right)+c_2\left(x,y,z\right)$$

where c_0, c_1, and c_2 are known functions.

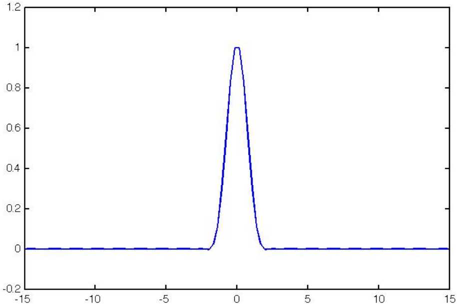
The two constants d_0 and d_1 are given by

$$\left(\begin{array}{c} d_0 \\ d_1 \end{array}\right)=-\left(\begin{array}{cc} c^+_{0yy} & c^+_{1yy} \\ c^+_{0yyy} & c^+_{1yyy} \end{array}\right)^{-1}\left(\begin{array}{c} c^+_{2yy} \\ c^+_{2yyy} \end{array}\right)$$

Almost-Localized forcing

Incident waves along the edge

$$\mathit{Incident}(x) \sim \sum_{\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]} \exp\left[-\lambda_0 \theta^2 + \mathrm{i} x \lambda_0 \sin\theta\right]$$



The width of the pulse becomes narrower for higher frequency

Response to a Localized excitation

Amplitude of simple harmonic oscillation : superposition of solutions